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GEOMETRICAL INEQUALITIES IN ACUTE TRIANGLES INVOLVING THE MEDIANS VII

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Abstract

The purpose of this paper is to give a negative answer to a new open question that aimed to generalize previous open questions formulated by researchers in the field of geometrical inequalities. In this sense we prove that from $a < b < c$ does not result $a^{\frac{p}{q}} + m_a^{\frac{p}{q}} < b^{\frac{p}{q}} + m_b^{\frac{p}{q}} < c^{\frac{p}{q}} + m_c^{\frac{p}{q}}$ in every acute triangle ABC, where $\frac{p}{q} \in \mathbb{Q}_+$. For the demonstration we deduce two propositions that allow the formulation of the main conclusion.

Keywords: geometrical inequalities, acute triangle, medians, bisectrices, altitudes

1 Introduction

Let us consider the acute triangle ABC with sides $a = BC, b = AC$ and $c = AB$. In [1] appeared the following open question due to Pál Erdős: "if ABC is an acute triangle such that $a < b < c$ then $a + l_a < b + l_b < c + l_c$", where l_a, l_b, l_c means the length of the interior bisectrices corresponding to the sides BC, AC and AB, respectively.

In [2] Mihály Bencze proposed the following open question, which is a generalization of the problem of Pál Erdős: "determine all points $M \in Int(ABC)$, for which in case of $BC < CA < AB$ we have $CB + AA' < CA + BB' < AB + CC'$, where A',B',C' is the intersection of AM, BM, CM with sides BC,CA,AB". Here with $Int(ABC)$ we denote the interior points of the triangle ABC.

If we try for "usual" acute triangles ABC, we can verify the validity of the Erdős inequality. But in [3] we realized to obtain an acute triangle for which the Erdős inequality is false: let ABC be such that $c = 10 + \epsilon, b = 10$ and $a = 1$, where $\epsilon > 0$ is a "very small" positive quantity. Using the trigonometrical way combined with some elementary properties from algebra and mathematical analysis we showed that for this "extreme" acute triangle from $a < b < c$ results $c + l_c < b + l_b$.

In [4] József Sándor proved some new geometrical inequalities using in the open question of Pál Erdős instead of bisectrices altitudes and medians.

In [5] Károly Dáné and Csaba Ignát studied the open question of Mihály Bencze using the computer.

In connection with Erdős's problem we formulated the following open question: "if ABC is an acute triangle such that $a < b < c$ then $a^2 + l_a^2 < b^2 + l_b^2 < c^2 + l_c^2$". In [6] we proved the validity of this statement.

At the same time we formulated another new open question: "if ABC is an acute triangle such that $a < b < c$ then $a^4 + l_a^4 < b^4 + l_b^4 < c^4 + l_c^4$". In [7] we realized to find two acute triangles ABC such that in the first triangle from $a < b < c$ ($b = a + \epsilon, c = a + 2\epsilon$ with $\epsilon > 0$ a small positive quantity) we obtained $a^4 + l_a^4 < b^4 + l_b^4$, but in the second triangle from $a < b < c$ ($b = a + \epsilon, c = a\sqrt{2}$ with $\epsilon > 0$ a small positive quantity) we deduced $a^4 + l_a^4 > b^4 + l_b^4$. So the answer to our question is negative.

Next we denote by h_a, h_b and h_c the length of the altitudes of the triangle ABC, which correspond to the sides BC,CA and AB, respectively. Then we can formulate the following more general similar question, replacing in the open question of Pál Erdős the bisectrices with altitudes: “if ABC is an acute triangle such that $a < b < c$ then $a^\alpha + h_a^\alpha < b^\alpha + h_b^\alpha < c^\alpha + h_c^\alpha$, where $\alpha \in \mathbb{R}$ is a real number". In [4] or [8] there is showed, that this property is true for all $\alpha \in \mathbb{R}^*$, where $\mathbb{R}^* = \mathbb{R} - \{0\}$.

Next we denote by m_a, m_b and m_c the length of the medians, which correspond to the sides BC, CA and AB, respectively. Then we can formulate the following more general similar question, replacing in the open question of Pál Erdős the bisectrices with medians: "if ABC is an acute triangle from $a < b < c$ it results $a^\alpha + m_a^\alpha < b^\alpha + m_b^\alpha < c^\alpha + m_c^\alpha$, where $\alpha \in \mathbb{R}$".

We mention, that this property is not obvious, because $a < b \Leftrightarrow a^2 < b^2 \Leftrightarrow 2(a^2+c^2) - b^2 < 2(b^2+c^2) - a^2 \Leftrightarrow \sqrt{\frac{2(a^2+c^2)-b^2}{4}} < \sqrt{\frac{2(b^2+c^2)-a^2}{4}} \Leftrightarrow m_b < m_a$.

Similarly $b < c \Leftrightarrow m_b > m_c$.

If $\alpha = 0$, then is immediately that our problem is false.

The above proposed problem is solved in [8] for $\alpha \in \{1, 2, 4\}$ and we obtained that for $\alpha = 1$ it is false and for $\alpha = 2$ and $\alpha = 4$ it is true. In [9] we showed for $\alpha = 8$, that our question is false, and in [10] we proved that for $\alpha = -2$ our inequality is false, too. In [11] we obtained for $\alpha = 2n, n \in \mathbb{N}, n \geq 3$, even natural numbers that our statement is false. In [12] we showed for $\alpha = 2n+1, n \in \mathbb{N}, n \geq 2$ odd natural numbers that our affirmation is false. In [13] we proved for $\alpha = -2n, n \in \mathbb{N}^*$, negative even integer numbers, that our statement is not valid. In [14] we proved for $\alpha = -2n-1, n \in \mathbb{N}$, negative even integer numbers, that our statement is not valid.

2 Main part

The purpose of this paper is to study this open question for $\alpha = \frac{p}{q}, \frac{p}{q} \in \mathbb{Q}, \frac{p}{q} > 0$ positive rational number, where $p, q \in \mathbb{N}$ and $q \neq 0$. If $q = 1$ then $\alpha = p \in \mathbb{N}^*$ and using the results from [11] and [12] we can deduce that for $p \geq 5$ our result is false. Next let $q \geq 2$ be a natural number.

Proposition 1. *There exist acute triangles ABC with $a < b < c$, such that for these triangles we have $a^{\frac{p}{q}} + (m_a)^{\frac{p}{q}} < b^{\frac{p}{q}} + (m_b)^{\frac{p}{q}}$ for every positive rational number $\frac{p}{q}$.*

Proof. We have the following sequence of equivalent inequalities: $a^{\frac{p}{q}} + (m_a)^{\frac{p}{q}} < b^{\frac{p}{q}} + (m_b)^{\frac{p}{q}} \Leftrightarrow (m_a)^{\frac{p}{q}} - (m_b)^{\frac{p}{q}} < b^{\frac{p}{q}} - a^{\frac{p}{q}} \Leftrightarrow (\sqrt[q]{m_a})^p - (\sqrt[q]{m_b})^p < (\sqrt[q]{a})^p - (\sqrt[q]{b})^p \Leftrightarrow (\sqrt[q]{m_a} - \sqrt[q]{m_b}) \cdot [(\sqrt[q]{m_a})^{p-1} + (\sqrt[q]{m_a})^{p-2} \cdot \sqrt[q]{m_b} + \cdots + (\sqrt[q]{m_a}) \cdot (\sqrt[q]{m_b})^{p-2} + (\sqrt[q]{m_b})^{p-1}] < (\sqrt[q]{a} - \sqrt[q]{b}) \cdot [(\sqrt[q]{a})^{p-1} + (\sqrt[q]{a})^{p-2} \cdot \sqrt[q]{b} + \cdots + (\sqrt[q]{a}) \cdot (\sqrt[q]{b})^{p-2} + (\sqrt[q]{b})^{p-1}] \Leftrightarrow$

$\Leftrightarrow (m_a - m_b) \cdot \frac{[(\sqrt[q]{m_a})^{p-1}+(\sqrt[q]{m_a})^{p-2}\cdot\sqrt[q]{m_b}+\cdots+(\sqrt[q]{m_a})\cdot(\sqrt[q]{m_b})^{p-2}+(\sqrt[q]{m_b})^{p-1}]}{[(\sqrt[q]{m_a})^{q-1}+(\sqrt[q]{m_a})^{q-2}\cdot\sqrt[q]{m_b}+\cdots+(\sqrt[q]{m_a})\cdot(\sqrt[q]{m_b})^{q-2}+(\sqrt[q]{m_b})^{q-1}]}$
$< (a-b) \cdot \frac{[(\sqrt[q]{a})^{p-1}+(\sqrt[q]{a})^{p-2}\cdot\sqrt[q]{b}+\cdots+(\sqrt[q]{a})\cdot(\sqrt[q]{b})^{p-2}+(\sqrt[q]{b})^{p-1}]}{[(\sqrt[q]{a})^{q-1}+(\sqrt[q]{a})^{q-2}\cdot\sqrt[q]{b}+\cdots+(\sqrt[q]{a})\cdot(\sqrt[q]{b})^{q-2}+(\sqrt[q]{b})^{q-1}]} \Leftrightarrow$
$\Leftrightarrow \frac{m_a^2 - m_b^2}{m_a + m_b} \cdot \frac{[(\sqrt[q]{m_a})^{p-1}+(\sqrt[q]{m_a})^{p-2}\cdot\sqrt[q]{m_b}+\cdots+(\sqrt[q]{m_a})\cdot(\sqrt[q]{m_b})^{p-2}+(\sqrt[q]{m_b})^{p-1}]}{[(\sqrt[q]{m_a})^{q-1}+(\sqrt[q]{m_a})^{q-2}\cdot\sqrt[q]{m_b}+\cdots+(\sqrt[q]{m_a})\cdot(\sqrt[q]{m_b})^{q-2}+(\sqrt[q]{m_b})^{q-1}]}$
$< \frac{a^2-b^2}{a+b} \cdot \frac{[(\sqrt[q]{a})^{p-1}+(\sqrt[q]{a})^{p-2}\cdot\sqrt[q]{b}+\cdots+(\sqrt[q]{a})\cdot(\sqrt[q]{b})^{p-2}+(\sqrt[q]{b})^{p-1}]}{[(\sqrt[q]{a})^{q-1}+(\sqrt[q]{a})^{q-2}\cdot\sqrt[q]{b}+\cdots+(\sqrt[q]{a})\cdot(\sqrt[q]{b})^{q-2}+(\sqrt[q]{b})^{q-1}]}$.

But $m_a^2 = \frac{2(b^2+c^2)-a^2}{4}$ and $m_b^2 = \frac{2(a^2+c^2)-b^2}{4}$, so $m_a^2 - m_b^2 = \frac{3\cdot(b^2-a^2)}{4} > 0$. This means, that

$a^{\frac{p}{q}} + (m_a)^{\frac{p}{q}} < b^{\frac{p}{q}} + (m_b)^{\frac{p}{q}} \Leftrightarrow \frac{3}{4\cdot(m_a+m_b)} \cdot \frac{[(\sqrt[q]{m_a})^{p-1}+(\sqrt[q]{m_a})^{p-2}\cdot\sqrt[q]{m_b}+\cdots+(\sqrt[q]{m_a})\cdot(\sqrt[q]{m_b})^{p-2}+(\sqrt[q]{m_b})^{p-1}]}{[(\sqrt[q]{m_a})^{q-1}+(\sqrt[q]{m_a})^{q-2}\cdot\sqrt[q]{m_b}+\cdots+(\sqrt[q]{m_a})\cdot(\sqrt[q]{m_b})^{q-2}+(\sqrt[q]{m_b})^{q-1}]}$
$< \frac{1}{a+b} \cdot \frac{[(\sqrt[q]{a})^{p-1}+(\sqrt[q]{a})^{p-2}\cdot\sqrt[q]{b}+\cdots+(\sqrt[q]{a})\cdot(\sqrt[q]{b})^{p-2}+(\sqrt[q]{b})^{p-1}]}{[(\sqrt[q]{a})^{q-1}+(\sqrt[q]{a})^{q-2}\cdot\sqrt[q]{b}+\cdots+(\sqrt[q]{a})\cdot(\sqrt[q]{b})^{q-2}+(\sqrt[q]{b})^{q-1}]}$.

At the first we choose the acute triangle ABC, such that $b = a + \epsilon$ and $c = a + 2\epsilon$, where $\epsilon > 0$ is a small positive quantity. Then

$$\begin{aligned}
\lim_{\epsilon\to 0} m_a^2 &= \lim_{\epsilon\to 0} \frac{2(b^2+c^2)-a^2}{4} \\
&= \lim_{\epsilon\to 0} \frac{2[(a+\epsilon)^2+(a+2\epsilon)^2]-a^2}{4} = \frac{3a^2}{4}, \\
\lim_{\epsilon\to 0} m_b^2 &= \lim_{\epsilon\to 0} \frac{2(a^2+c^2)-b^2}{4} \\
&= \lim_{\epsilon\to 0} \frac{2[a^2+(a+2\epsilon)^2]-(a+\epsilon)^2}{4} = \frac{3a^2}{4}, \\
\lim_{\epsilon\to 0} b &= \lim_{\epsilon\to 0}(a+\epsilon) = a.
\end{aligned}$$

This means, that $\lim_{\epsilon\to 0} \frac{3}{4\cdot(m_a+m_b)} \cdot \frac{[(\sqrt[q]{m_a})^{p-1}+(\sqrt[q]{m_a})^{p-2}\cdot\sqrt[q]{m_b}+\cdots+(\sqrt[q]{m_a})\cdot(\sqrt[q]{m_b})^{p-2}+(\sqrt[q]{m_b})^{p-1}]}{[(\sqrt[q]{m_a})^{q-1}+(\sqrt[q]{m_a})^{q-2}\cdot\sqrt[q]{m_b}+\cdots+(\sqrt[q]{m_a})\cdot(\sqrt[q]{m_b})^{q-2}+(\sqrt[q]{m_b})^{q-1}]}$
$\leq \lim_{\epsilon\to 0} \frac{1}{a+b} \cdot \frac{[(\sqrt[q]{a})^{p-1}+(\sqrt[q]{a})^{p-2}\cdot\sqrt[q]{b}+\cdots+(\sqrt[q]{a})\cdot(\sqrt[q]{b})^{p-2}+(\sqrt[q]{b})^{p-1}]}{[(\sqrt[q]{a})^{q-1}+(\sqrt[q]{a})^{q-2}\cdot\sqrt[q]{b}+\cdots+(\sqrt[q]{a})\cdot(\sqrt[q]{b})^{q-2}+(\sqrt[q]{b})^{q-1}]}$

i.e. $\frac{3}{4} \cdot \frac{1}{2\cdot\frac{\sqrt{3}}{2}\cdot a} \cdot \frac{(\frac{\sqrt{3}}{2}\cdot a)^{\frac{p-1}{q}}\cdot p}{(\frac{\sqrt{3}}{2}a)^{\frac{q-1}{q}}\cdot q} \leq \frac{1}{2\cdot a} \cdot \frac{a^{\frac{p-1}{q}}\cdot p}{a^{\frac{q-1}{q}}\cdot q}$, i.e.

$\frac{3}{4} \cdot \frac{1}{\frac{\sqrt{3}}{2}} \cdot \frac{(\frac{\sqrt{3}}{2})^{\frac{p-1}{q}}}{(\frac{\sqrt{3}}{2})^{\frac{q-1}{q}}} \leq 1$ so $(\frac{\sqrt{3}}{2})^{\frac{p}{q}} \leq 1$.

We can see immediately that the inequality $(\frac{\sqrt{3}}{2})^{2n+1} < 1$ is true for all $n \in \mathbb{N}$. Using the definition of the limit we can conclude, that there exists a small positive real number $\epsilon_0 > 0$ such that for the triangle $A_0B_0C_0$ with $B_0C_0 = a, A_0C_0 = b = a + \epsilon_0$ and $A_0B_0 = c = a + 2\epsilon_0$ we obtain $a^{\frac{p}{q}} + m_a^{\frac{p}{q}} < b^{\frac{p}{q}} + m_b^{\frac{p}{q}}$. □

Proposition 2. *There exist acute triangles ABC with $a < b < c$, such that for these triangles we have $a^{\frac{p}{q}} + (m_a)^{\frac{p}{q}} > b^{\frac{p}{q}} + (m_b)^{\frac{p}{q}}$ for every $\frac{p}{q} > \log_{\frac{5}{4}} (\frac{4}{3})^2$.*

Proof. Using the above presented sequence of ideas we get similarly, that $a^{\frac{p}{q}} + (m_a)^{\frac{p}{q}} > b^{\frac{p}{q}} + (m_b)^{\frac{p}{q}} \Leftrightarrow \Leftrightarrow \frac{3}{4\cdot(m_a+m_b)} \cdot \frac{[(\sqrt[q]{m_a})^{p-1}+(\sqrt[q]{m_a})^{p-2}\cdot\sqrt[q]{m_b}+\cdots+(\sqrt[q]{m_a})\cdot(\sqrt[q]{m_b})^{p-2}+(\sqrt[q]{m_b})^{p-1}]}{[(\sqrt[q]{m_a})^{q-1}+(\sqrt[q]{m_a})^{q-2}\cdot\sqrt[q]{m_b}+\cdots+(\sqrt[q]{m_a})\cdot(\sqrt[q]{m_b})^{q-2}+(\sqrt[q]{m_b})^{q-1}]}$

$> \frac{1}{a+b} \cdot \frac{[(\sqrt[q]{a})^{p-1}+(\sqrt[q]{a})^{p-2}\cdot\sqrt[q]{b}+\cdots+(\sqrt[q]{a})\cdot(\sqrt[q]{b})^{p-2}+(\sqrt[q]{b})^{p-1}]}{[(\sqrt[q]{a})^{q-1}+(\sqrt[q]{a})^{q-2}\cdot\sqrt[q]{b}+\cdots+(\sqrt[q]{a})\cdot(\sqrt[q]{b})^{q-2}+(\sqrt[q]{b})^{q-1}]}.$

At the second we choose the acute triangle ABC, such that $b = a + \epsilon$ and $c = a\sqrt{2}$, where $\epsilon > 0$ is a small positive quantity. Then

$$\begin{aligned}\lim_{\epsilon\to 0} m_a^2 &= \lim_{\epsilon\to 0} \frac{2(b^2+c^2)-a^2}{4}\\ &= \lim_{\epsilon\to 0} \frac{2[(a+\epsilon)^2+2a^2]-a^2}{4} = \frac{5a^2}{4},\\ \lim_{\epsilon\to 0} m_b^2 &= \lim_{\epsilon\to 0} \frac{2(a^2+c^2)-b^2}{4}\\ &= \lim_{\epsilon\to 0} \frac{2[a^2+2a^2]-(a+\epsilon)^2}{4} = \frac{5a^2}{4}, \text{ and}\\ \lim_{\epsilon\to 0} b &= \lim_{\epsilon\to 0}(a+\epsilon) = a.\end{aligned}$$

This means, that $\lim_{\epsilon\to 0} \frac{3}{4\cdot(m_a+m_b)} \cdot \frac{[(\sqrt[q]{m_a})^{p-1}+(\sqrt[q]{m_a})^{p-2}\cdot\sqrt[q]{m_b}+\cdots+(\sqrt[q]{m_a})\cdot(\sqrt[q]{m_b})^{p-2}+(\sqrt[q]{m_b})^{p-1}]}{[(\sqrt[q]{m_a})^{q-1}+(\sqrt[q]{m_a})^{q-2}\cdot\sqrt[q]{m_b}+\cdots+(\sqrt[q]{m_a})\cdot(\sqrt[q]{m_b})^{q-2}+(\sqrt[q]{m_b})^{q-1}]}$ $\geq \lim_{\epsilon\to 0} \frac{1}{a+b} \cdot \frac{[(\sqrt[q]{a})^{p-1}+(\sqrt[q]{a})^{p-2}\cdot\sqrt[q]{b}+\cdots+(\sqrt[q]{a})\cdot(\sqrt[q]{b})^{p-2}+(\sqrt[q]{b})^{p-1}]}{[(\sqrt[q]{a})^{q-1}+(\sqrt[q]{a})^{q-2}\cdot\sqrt[q]{b}+\cdots+(\sqrt[q]{a})\cdot(\sqrt[q]{b})^{q-2}+(\sqrt[q]{b})^{q-1}]}$
i.e. $\frac{3}{4} \cdot \frac{1}{2\cdot\frac{\sqrt{5}}{2}\cdot a} \cdot \frac{(\frac{\sqrt{5}}{2}\cdot a)^{\frac{p-1}{q}}\cdot p}{(\frac{\sqrt{5}}{2}a)^{\frac{q-1}{q}}\cdot q} \geq \frac{1}{2\cdot a} \cdot \frac{a^{\frac{p-1}{q}}\cdot p}{a^{\frac{q-1}{q}}\cdot q}$,
i.e. $\frac{3}{4} \cdot \frac{1}{\frac{\sqrt{5}}{2}} \cdot \frac{(\frac{\sqrt{5}}{2})^{\frac{p-1}{q}}}{(\frac{\sqrt{5}}{2})^{\frac{q-1}{q}}} \geq 1$ i.e. $(\frac{\sqrt{5}}{2})^{\frac{p}{q}} \geq \frac{4}{3}$. i.e. $(\frac{5}{4})^{\frac{p}{q}} \geq (\frac{4}{3})^2$.

But we can see immediately that the inequality $(\frac{5}{4})^{\frac{p}{q}} > (\frac{4}{3})^2$ is true for all $\frac{p}{q} > \log_{\frac{5}{4}}(\frac{4}{3})^2$. Using the definition of the limit we can conclude, that there exists a small positive real number $\epsilon_1 > 0$ such that for the triangle $A_1B_1C_1$ with $B_1C_1 = a, A_1C_1 = b = a+\epsilon_1$ and $A_1B_1 = c = a\sqrt{2}$ we obtain $a^{\frac{p}{q}}+(m_a)^{\frac{p}{q}} > b^{\frac{p}{q}} + (m_b)^{\frac{p}{q}}$ for every $\frac{p}{q} > \log_{\frac{5}{4}}(\frac{4}{3})^2$. $\square$

3 Discussion and conclusion

Conclusion Let $\alpha = \frac{p}{q} \in \mathbb{Q}$, $\frac{p}{q} > 0$ be a positive rational number. If $q = 1$ then for $\alpha = p \in \mathbb{N}^*, p \geq 5$ our inequality is false. If $q \geq 2, q \in \mathbb{N}$ then for $\alpha = \frac{p}{q} > \log_{\frac{5}{4}}(\frac{4}{3})^2$ from the propositions we can deduce that our open question is not true. Because $5 > \log_{\frac{5}{4}}(\frac{4}{3})^2 > 4$ we can conclude that our open question is false for every positive rational number $\alpha = \frac{p}{q} \in \mathbb{Q}$ with $\alpha = \frac{p}{q} > \log_{\frac{5}{4}}(\frac{4}{3})^2$.

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